8.5 Equivalence Relations
8.6 Partial Orderings

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§8.5: Equivalence Relations

- An *equivalence relation* (e.r.) on a set A is simply any binary relation on A that is reflexive, symmetric, and transitive.
  - *E.g.*, = itself is an equivalence relation.
  - For any function $f{:}A\rightarrow B$, the relation "have the same f value", or $=_f :\equiv \{(a_1,a_2) \mid f(a_1)=f(a_2)\}$ is an equivalence relation,
    - *e.g.*, let m = "mother of" then $=_m$ = "have the same mother" is an e.r.

Equivalence Relation Examples

- "Strings *a* and *b* are the same length."
- "Integers *a* and *b* have the same absolute value."
- "Real numbers *a* and *b* have the same fractional part." (*i.e.*, $a - b \in \mathbf{Z}$)
- "Integers *a* and *b* have the same residue modulo *m*." (for a given $m>1$)

Equivalence Classes

- Let R be any equiv. rel. on a set A.
- The *equivalence class* of a,
  $[a]_R :\equiv \{ b \mid aRb \}$ (optional subscript R)
  - It is the set of all elements of A that are "equivalent" to a according to the eq.rel. R.
  - Each such b (including a itself) is called a *representative* of $[a]_R$.
- Since $f(a)=[a]_R$ is a function of a, any equivalence relation R can be defined using $aRb :\equiv$ "a and b have the same f value", given f.

Equivalence Class Examples

- "Strings a and b are the same length."
  - $[a]$ = the set of all strings of the same length as a.
- "Integers a and b have the same absolute value."
  - $[a]$ = the set $\{a, -a\}$
- "Real numbers a and b have the same fractional part (*i.e.*, $a - b \in \mathbf{Z}$)."
  - $[a]$ = the set $\{\ldots, a-2, a-1, a, a+1, a+2, \ldots\}$
- "Integers a and b have the same residue modulo m." (for a given $m>1$)
  - $[a]$ = the set $\{\ldots, a-2m, a-m, a, a+m, a+2m, \ldots\}$

§8.6: Partial Orderings

- A relation R on A is called a *partial ordering* or *partial order* iff it is reflexive, antisymmetric, and transitive.
  - We often use a symbol looking something like $\preccurlyeq$ (or analogous shapes) for such relations.
  - Examples: $\leq$, $\geq$ on real numbers, $\subseteq$, $\supseteq$ on sets.
  - Another example: the divides relation $|$ on $\mathbf{Z}^+$.
    - Note it is not necessarily the case that either $a \preccurlyeq b$ or $b \preccurlyeq a$.
- A set A together with a partial order $\preccurlyeq$ on A is called a *partially ordered set* or *poset* and is denoted $(A, \preccurlyeq)$.