8.3 Representing Relations

Rosen 6th ed., Ch. 8

§8.3: Representing Relations

- Some ways to represent *n*-ary relations:
  - With an explicit list or table of its tuples.
  - With a function from the domain to **{T,F}**.
    - Or with an algorithm for computing this function.
- Some special ways to represent binary relations:
  - With a zero-one matrix.
  - With a directed graph.

Using Zero-One Matrices

- To represent a binary relation $R{:}A\times B$ by an $|A|\times|B|$ 0-1 matrix $\mathbf{M}_R = [m_{ij}]$, let $m_{ij} = 1$ iff $(a_i,b_j)\in R$.
- *E.g.*, Suppose Joe likes Susan and Mary, Fred likes Mary, and Mark likes Sally.
- Then the 0-1 matrix representation of the relation Likes:Boys×Girls relation is:

$$\begin{array}{r c} & \begin{array}{ccc} \text{Susan} & \text{Mary} & \text{Sally} \end{array} \\ \begin{array}{r} \text{Joe} \\ \text{Fred} \\ \text{Mark} \end{array} & \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

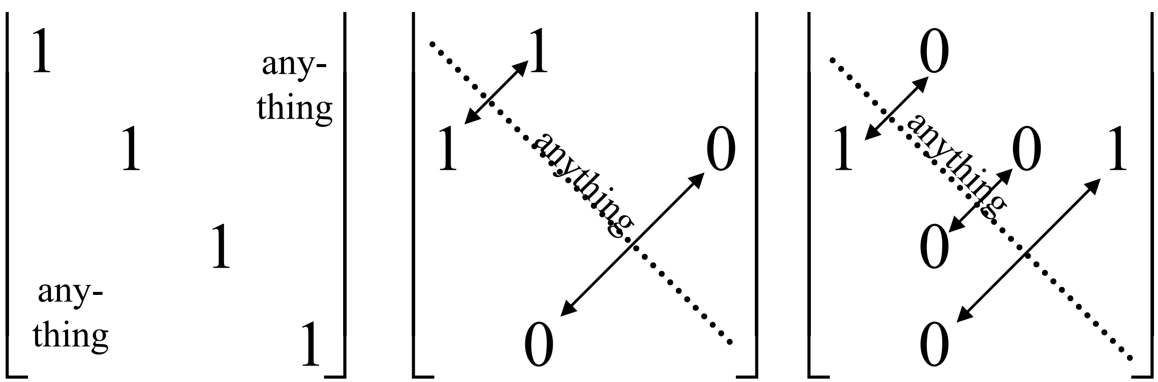
Properties of Relations

- Reflexivity: A relation R on A *x* A is reflexive if for all *a*□A, (*a*,*a*) □R.
- Symmetry: A relation R on AxA is symmetric if (*x*,*y*) □ R implies (*y*,*x*) □ R.
- Anti-symmetry:

  A relation on A *x* A is anti-symmetric if (*a*,*b*) □ R implies (*b*,*a*) □ R. Or a = b.

Zero-One Reflexive, Symmetric

- Terms: *Reflexive*, *symmetric*, *and antisymmetric*.
  - These relation characteristics are very easy to recognize by inspection of the zero-one matrix.



Reflexive:
all 1's on diagonal

Symmetric:
all identical
across diagonal

Antisymmetric:
all 1's are across
from 0's

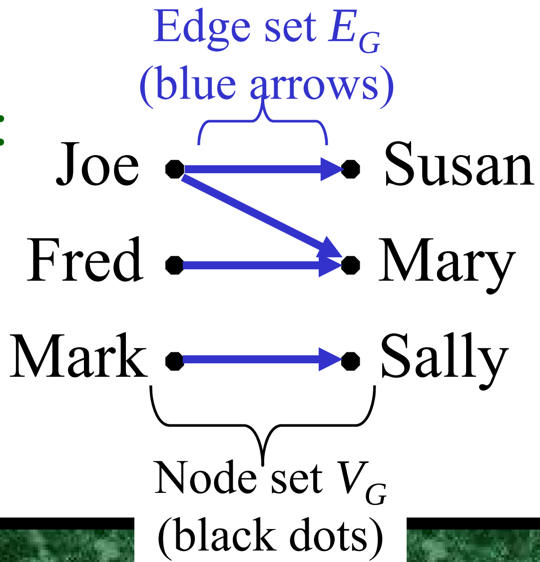
Using Directed Graphs

- A *directed graph* or *digraph* $G=(V_G,E_G)$ is a set V_G of *vertices (nodes)* with a set $E_G \subseteq V_G \times V_G$ of *edges (arcs,links)*. Visually represented using dots for nodes, and arrows for edges. Notice that a relation $R{:}A \times B$ can be represented as a graph $G_R=(V_G=A \cup B,\ E_G=R)$.

Matrix representation $\mathbf{M}_R$:

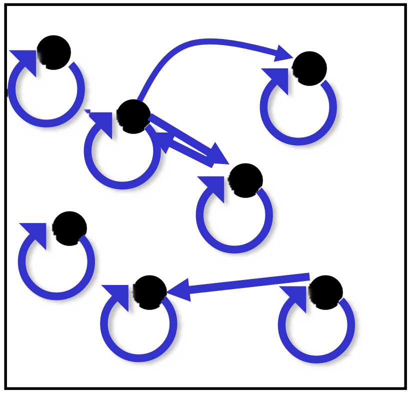
$$
\begin{array}{c}
 \\ \text{Joe} \\ \text{Fred} \\ \text{Mark}
\end{array}
\begin{array}{c}
\begin{array}{ccc} \text{Susan} & \text{Mary} & \text{Sally} \end{array} \\
\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\end{array}
$$

Graph rep. G_R:

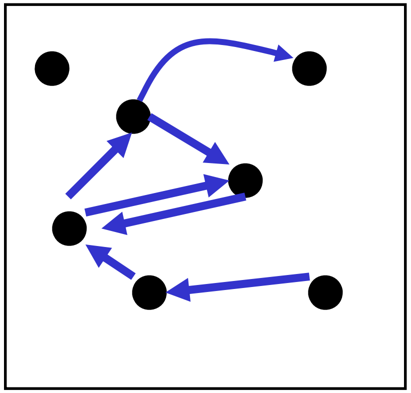


Digraph Reflexive, Symmetric

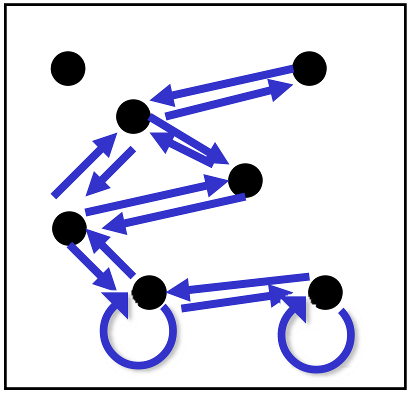
It is extremely easy to recognize the reflexive/irreflexive/ symmetric/antisymmetric properties by graph inspection.



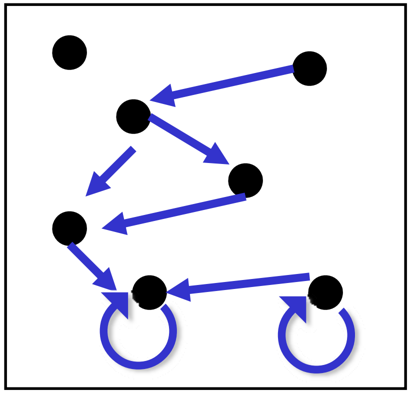
Reflexive:
Every node has a self-loop



Irreflexive:
No node links to itself



Symmetric:
Every link is bidirectional



Antisymmetric:
No link is bidirectional

These are asymmetric & non-antisymmetric

These are non-reflexive & non-irreflexive